Prof. Kincade

March 28 – Section 6.5 {Properties of Logarithms

• Definition of a logarithm; {Also called a "coupling formula"

→ $\log_b x = y \Rightarrow b^y = x$

Transform a log... into an exponent

→ $\log_a(1) = 0$ {The log base a of 1 is always 0; $a^0 = 1$

→ $\log_a(a) = 1$ {The log base a of itself is 1; $a^1 = a$

→ $\log_a(m \cdot n) = \log_a m + \log_a n$ {The log of a product is the sum of logs of the factors

→ $\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$ {The log of a quotient is the difference of logs of the factors

→ $\log_a b = \frac{\log_c b}{\log_c a}$ {Change of base formula; this is useful to transform a log into a more useful form

→ $\log_a m^r = r \cdot \log_a m$

→ If $m = n$, then $\log_b m = \log_b n$ {This says we can apply logs to an equation, so long as we do this to both sides

→ $a^{\log_a m} = \boxed{m}$ {logs and exponents are inverses. If you have a base 'a', with a log 'a', their inverse nature acts as a cancellation, and you are left with your argument, 'm', the thing you are taking the log of.

14, homework:

$\log_2 2^{-13} = -13 \cdot \log_2 2 = -13 \cdot 1 = \boxed{-13}$

($\log_2 2$ = 1 !)

16) $\ln e^{\sqrt{2}} \Rightarrow \log_e e^{\sqrt{2}} = \sqrt{2} \cdot \log_e e = \boxed{\sqrt{2}}$

($\ln$ = $\log_e$)

Inverse Cancellation!

18) $e^{\ln 8} = e^{\log_e 8} = \boxed{8}$

$\Rightarrow e^{2 \cdot \ln 8} = e^{2 \log_e 8}$ → $2 \log_e 8 = \log_e 8^2$ ☆ → $= 8^2 = \boxed{64}$

☆ $e^{\log_e \sqrt{5}} = \boxed{\sqrt{5}}$

☆ $e^{\log_e 4^2} = 4^2 = \boxed{16}$

Product Rule

20) $\log_6 9 + \log_6 4 = \log_6 (36) = \boxed{2}$ $\{ 6^2 = 36$

($36 = 9 \cdot 4$)

Quotient rule

22) $\log_8 16 - \log_8 2 = \log_8 (16/2) = \log_8 (8) = \boxed{1}$

24) $\log_3 8 \cdot \log_8 9 = \frac{\log_{10} 8}{\log_{10} 3} \cdot \frac{\log_{10} 9}{\log_{10} 8} = \frac{\log 9}{\log 3} = \frac{\log 3^2}{\log 3}$

$\Rightarrow \frac{2 \log 3}{\log 3} = \boxed{2}$

26) $5^{\log_5 6 + \log_5 7} = 5^{\log_5 (42)} = \boxed{42}$

↳ $(5^{\log_5 6})(5^{\log_5 7}) = (6)(7) =$ → 42

28) $e^{\log_e \sqrt{9}} = \sqrt{9} = 3$

---

30) $\ln 2 = a, \quad \ln 3 = b$

$\ln (2/3)$

$\rightarrow \ln 2 - \ln 3 \rightarrow a - b$

32) $\ln(0.5) = \ln\left(\frac{1}{2}\right) = \ln 1 - \ln 2$

$0 \; \boxed{-a}$

- Expand / Contract Logs

$\log\left(\frac{x^2 \cdot y^{1/3}}{z^{3/4} \cdot w^5}\right) = \log(x^2 \cdot y^{1/3}) - \log(z^{3/4} \cdot w^5)$

$= \log(x^2) + \log(y^{1/3}) - [\log(z^{3/4}) + \log(w^5)]$

distribute the negative!

$= 2\log x + \frac{1}{3}\log y - \frac{3}{4}\log z - 5\log w$

$\log_2\left(\frac{\sqrt{x} \cdot \sqrt[3]{y} \cdot z^4}{w^{3/5} \cdot k^{5/8}}\right) = \log_2(\sqrt{x}) + \log_2(\sqrt[3]{y}) + \log_2(z^4) - [\log_2(w^{3/5}) + \log_2(k^{5/8})]$

Quotient & product rules

rewriting radicals as exponents will force an exponent to appear!

$= \log_2(x^{1/2}) + \log_2(y^{1/3}) + \log_2(z^4) - \log_2(w^{3/5}) - \log_2(k^{5/8})$

$= \frac{1}{2}\log_2 x + \frac{1}{3}\log_2 y + 4\log_2 z - 3/5\log_2 w - 5/8\log_2(k)$

which we can then bring down as a coefficient

→ Rewrite as a single logarithm

- $4\ln x - 6\ln y + \frac{1}{3}\ln z - \frac{2}{3}\ln k$

$\ln x^4 - \ln y^6 + \ln\sqrt[3]{z} - \ln\sqrt[3]{k^2}$

$\ln\left(\frac{x^4 \cdot \sqrt[3]{z}}{y^6\,\sqrt[3]{k^2}}\right)$

→ Section 6.3 → exponential equations with like bases:

76) $9^{2x} \cdot 27^{x^2} = 3^{-1}$

$((3)^2)^{2x} \cdot ((3)^3)^{x^2} = 3^{-1}$

$(3)^{4x} \cdot (3)^{3x^2} = (3)^{-1}$

$(3)^{3x^2+4x} = (3)^{-1}$

$4x + 3x^2 = -1$

$3x^2 + 4x + 1 = 0$

$(3x+1)(x+1) \Rightarrow 3x = -1 \Rightarrow x = -1/3$

$3x = -1 \Rightarrow x = -1/3$

$2^{3x} = 3^{2x-1}$ ← Equations w/ unlike bases

☆ remember $m = N \Rightarrow \log m = \log N$

$\ln 2^{3x} = \ln 3^{2x-1} \Rightarrow (3x) \cdot \ln 2 = (2x-1) \ln 3$

decimal #'s!

$\Rightarrow x(3 \ln 2) = x(2 \ln 3) - \ln 3$

$\Rightarrow x(3 \ln 2) - x(2 \ln 3) = -\ln 3$

$x[3 \ln 2 - 2 \ln 3] = -\ln 3$

$x[\ln 2^3 - \ln 3^2] = -\ln 3$

$x[\ln 8 - \ln 9] = -\ln 3$

$x[\ln(\frac{8}{9})] = -\ln 3$

☆ $x = \dfrac{-\ln 3}{\ln(8/9)}$

$(3x)(\text{decimal}) = (2x-1)(\text{decimal})$

$3x(0.6931) = (2x-1)(1.0986)$